

**Spring 2026: Mathematical Statistics**  
**Recitation Worksheet 8**  
 Mar. 27, 2026

1. True or False:

- (a) The generalized likelihood ratio statistic  $LR$  is always less than or equal to 1.
- (b) Since  $\text{Var}(F_n(x)) = S_n^2(x) = \frac{F(x)(1-F(x))}{n}$ ,

$$\left( F_n - 1.96 \frac{1}{\sqrt{n}} S_n(x), F_n + 1.96 \frac{1}{\sqrt{n}} S_n(x) \right)$$

is a 0.95 confidence interval for the true CDF  $F(x)$ .

2. [9.35] Under a standard genetic model, the genotypes  $AA$ ,  $Aa$ , and  $aa$  occur with probabilities  $(1 - \theta)^2$ ,  $2\theta(1 - \theta)$ , and  $\theta^2$ , respectively, for some  $0 \leq \theta \leq 1$ . A sample of 190 people reveals that 10 have type  $AA$ , 68 have type  $Aa$ , and 112 have type  $aa$ . [You may use the fact that the MLE is  $\hat{\theta} = 0.768$ . You should be able to derive this yourself to check it.]

- (a) What is the Pearson chi-squared statistic?
- (b) Use the table of the CDF of the  $\chi^2$  distribution on the other side of this page to estimate the  $p$ -value of the data. At a significance level of 0.05, would we accept or reject the null hypothesis that the data comes from our model?

3. Suppose we have two samples from a distribution,  $X_1 = 2$  and  $X_2 = 4$ .

- (a) Write out all the possible samples of size  $n = 2$  with replacement from this data.
- (b) Letting  $B$  be all possible samples, compute the  $B$  sample medians.
- (c) What is the bootstrap estimate of the variance of the sample median for this tiny dataset?

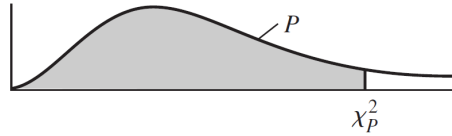
*Note: this is just an example for recitation, we would never do this in practice with so little data.*

4. A researcher claims that a specific random number generator produces values following a Uniform(0, 1) distribution. You generate a sample of  $n = 4$  independent observations to test this claim:

$$X = \{0.75, 0.40, 0.15, 0.90\}$$

- (a) State the null hypothesis  $H_0$  and the alternative hypothesis  $H_1$ .
- (b) Calculate and sketch the empirical CDF,  $F_n(x)$ , for this sample.
- (c) Calculate the K-S test statistic  $d_{KS} = \sup_x |F_n(x) - F(x)|$ .
- (d) At the  $\alpha = 0.05$  level, should you reject  $H_0$ ? Use the table on the other side of this page.
- (e) At the same level, should we reject  $H_0$  if  $X = \{0.82, 0.71, 0.96, 0.63\}$ ?

**TABLE 3** Percentiles of the  $\chi^2$  Distribution—Values of  $\chi^2_P$  Corresponding to  $P$



$df$	$\chi^2_{.005}$	$\chi^2_{.01}$	$\chi^2_{.025}$	$\chi^2_{.05}$	$\chi^2_{.10}$	$\chi^2_{.90}$	$\chi^2_{.95}$	$\chi^2_{.975}$	$\chi^2_{.99}$	$\chi^2_{.995}$
1	.000039	.00016	.00098	.0039	.0158	2.71	3.84	5.02	6.63	7.88
2	.0100	.0201	.0506	.1026	.2107	4.61	5.99	7.38	9.21	10.60
3	.0717	.115	.216	.352	.584	6.25	7.81	9.35	11.34	12.84
4	.207	.297	.484	.711	1.064	7.78	9.49	11.14	13.28	14.86
5	.412	.554	.831	1.15	1.61	9.24	11.07	12.83	15.09	16.75
6	.676	.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19

Kolmogorov distribution: table of  $K(\alpha)$  values for sample size  $n$

$n$	$\alpha$ 0.01	$\alpha$ 0.05	$\alpha$ 0.1	$\alpha$ 0.15	$\alpha$ 0.2
1	0.995	0.975	0.950	0.925	0.900
2	0.929	0.842	0.776	0.726	0.684
3	0.828	0.708	0.642	0.597	0.565
4	0.733	0.624	0.564	0.525	0.494
5	0.669	0.565	0.510	0.474	0.446
6	0.618	0.521	0.470	0.436	0.410
7	0.577	0.486	0.438	0.405	0.381
8	0.543	0.457	0.411	0.381	0.358
9	0.514	0.432	0.388	0.360	0.339
10	0.490	0.410	0.368	0.342	0.322