Spring 2026: Mathematical Statistics
Recitation Worksheet 7
March 13, 2026

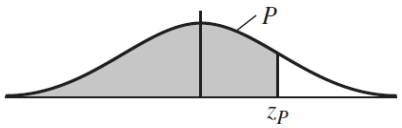
1. [9.5 and 9.23] True or False? Give a justification for your answer.
   (a) If the significance level of a test is decreased, the power will increase.
   (b) If a test is rejected at the significance level α, the probability that the null hypothesis is true equals α.
   (c) The probability that the null hypothesis is falsely rejected is equal to the power of the test.
   (d) The power of a test is determined by the null distribution of the test statistic.
   (e) The likelihood ratio is a random variable.
   (f) If the p-value is 0.03, the corresponding test will reject at the significance level 0.02.
   (g) Suppose that a 99% CI for the mean μ of a normal distribution is found to be $(-2.0, 3.0)$. A test of $H_0 : \mu = -3$ versus $H_A : \mu \neq -3$ would reject H_0 at the 0.01 significance level.

2. [9.19] Under H_0 a random variable has the CDF $F_0(x) = x^2, 0 \leq x \leq 1$, and under H_1 it has the CDF $F_1(x) = x^3, 0 \leq x \leq 1$.
   (a) What is the form of the likelihood ratio test of H_0 versus H_1?
   (b) What is the rejection region of a level α test?
   (c) What is the power of the test?

3. [9.20] Consider two probability density functions on $[0, 1] : f_0(x) = 1$, and $f_1(x) = 2x$. Among all the tests of the null hypothesis $H_0 : X \sim f_0(x)$ versus the alternative $H_1 : X \sim f_1(x)$, with significance level $\alpha = 0.10$, how large can the power possibly be?

4. [9.28] Suppose that a test statistic T has a standard normal null distribution.
   (a) If the test rejects for large values of $|T|$, what is the p-value corresponding to $T = 1.50$?
   (b) Answer the same question if the test rejects for large T.

TABLE 2 Cumulative Normal Distribution—Values of P Corresponding to z_p for the Normal Curve



z is the standard normal variable. The value of P for $-z_p$ equals 1 minus the value of P for $+z_p$; for example, the P for –1.62 equals 1 – .9474 = .0526.

z_p	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936