Spring 2026: Mathematical Statistics
Recitation Worksheet 6
Mar. 6, 2025

1. True or False? Let $\pi(\theta) = \text{Unif}([0,1])$ be a prior. The posterior mode of θ cannot fall outside of $[0,1]$.

2. [8.50, continued from HW] Let $X_1, \ldots, X_n$ be an i.i.d. sample from a Rayleigh distribution with parameter $\theta > 0$:
$$f(x \mid \theta) = \frac{x}{\theta^2} e^{-x^2/(2\theta^2)}, \qquad x \geq 0.$$
For this problem, you may use the fact that $\mathbb{E}[X_i^2] = 2\theta^2$.

   On Homework 5 you found the MLE
$$\hat{\theta}_{\text{MLE}} = \sqrt{\frac{1}{2n}\sum_{i=1}^{n} X_i^2}.$$
Now we are interested in the distribution of this estimator.

   (a) Find the asymptotic variance of the maximum likelihood estimator.

   (b) Find an approximate 95% confidence interval for θ.

3. [8.61] Laplace's rule of succession claims that when an event happens n times in a row and never fails to happen, the probability that the event will occur the next time is $(n+1)/(n+2)$. Show that this is the posterior mean of the probability θ of each event happening, given a certain simple choice of prior.

4. [8.60] Let $X_1, \ldots, X_n$ be an i.i.d. sample from an exponential distribution with the density function
$$f(x|\tau) = \frac{1}{\tau} e^{-x/\tau}, \quad x \geq 0.$$
   (a) Find the mle of τ.

   (b) Show that the mle is unbiased, and find its exact variance.

   (c) Is there any other unbiased estimate with smaller variance?

   (d) Find the form of an approximate confidence interval for τ.