Spring 2026: Mathematical Statistics
Recitation Worksheet 5
Feb. 27, 2026

1. [8.5] Suppose that X is a discrete random variable with $P(X=1)=\theta$ and $P(X=2)=1-\theta$. Three independent observations are made: $X_1=1, X_2=2$, and $X_3=2$.
   (a) Find the method of moments estimator of θ.
   (b) What is the likelihood function?
   (c) What is the maximum likelihood estimator of θ?

2. True or False?
   (a) MLE estimators are unbiased.
   (b) The square of the MLE estimator of a parameter θ converges to θ^2 in probability, for any parameter and any observations $X_1,\dots,X_n$.
   (c) The MOM estimator always exists.
   (d) The likelihood function integrates to 1, i.e.,
   $$\int_{-\infty}^{\infty} \mathcal{L}_n(\theta)d\theta = 1.$$

3. [8.21] Suppose that $X_1, X_2, \dots, X_n$ are i.i.d. with density
   $$f(x) = e^{-(x-\theta)}, \quad x \geq \theta,$$
   and $f(x)=0$ otherwise.
   (a) Find the method of moments estimate of θ.
   (b) Find the MLE of θ. (Hint: Be careful, and don't differentiate before thinking. For what values of θ is the likelihood positive?)

4. [8.16, expanded] Suppose $X_1,\dots,X_n$ are i.i.d. with density
   $$f(x \mid \sigma) = \frac{1}{2\sigma}\exp\left(-\frac{|x|}{\sigma}\right), \qquad \sigma > 0,$$
   which is the Laplace distribution centered at 0 with scale parameter σ. [Hint: you may use the fact that $\int_0^\infty x^n e^{-ax}dx = \frac{n!}{a^{n+1}}$.]
   (a) Find the regular method of moments estimator of σ. (How many moments do you need to calculate?)
   (b) Find the generalized method of moments estimator of σ using $g(X)=|X|$.
   (c) Find the MLE of σ.