Spring 2026: Mathematical Statistics
Recitation 1 Problems
Jan. 30, 2026

1. [1.69 and 1.70] True or False?
   (a) If A and B are disjoint, can they be independent?
   (b) If $A \subset B$, can A and B be independent?
2. [2.56] If $X \sim N(0, \sigma^2)$, find the density of $Y = |X|$.
3. [3.18] Let X and Y have the joint density function
$$f(x,y) = \begin{cases} k(x-y) & 0 \le y \le x \le 1 \\ 0 & \text{otherwise.} \end{cases}$$
   (a) Sketch the region over which the density is positive.
   (b) Find k. (Why can we determine k from the information given?)
   (c) Find the marginal density of X.
   (d) Find the conditional density of Y given X.
4. [3.31] Suppose that (X, Y) is uniform on the disk of radius 1. Without doing any calculations, argue that X and Y are not independent.
5. [4.31] Let X be uniformly distributed on the interval $[1, 2]$. Find $\mathbb{E}\left[\frac{1}{X}\right]$. Is $\mathbb{E}\left[\frac{1}{X}\right] = \frac{1}{\mathbb{E}X}$?
6. Suppose I offer you a game where a fair coin is tossed repeatedly. Let X be the number of tosses until the first head appears (e.g., if the first toss is a head, $X = 1$). If $X = k$, you win 2^k dollars. How much should you pay to play a round of this game?