Spring 2026: Mathematical Statistics
Recitation Worksheet 10
April 10, 2026

1. [11.21] A study was done to compare the performances of engine bearings made of different compounds. Ten bearings of each type were tested. The following table gives the times until failure (in units of millions of cycles):

Type I	Type II
3.03	3.19
5.53	4.26
5.60	4.47
9.30	4.53
9.92	4.67
12.51	4.69
12.95	12.78
15.21	6.79
16.04	9.37
16.84	12.75

   To avoid boring arithmetic, you may use the following facts (where Type I are the X_i's and Type II are the Y_i's):

$$\bar{X} = 10.69 \qquad \bar{Y} = 6.75$$
$$\sum (X_i - \bar{X})^2 = 209.03 \qquad \sum (Y_i - \bar{Y})^2 = 116.80.$$

   (a) Test the hypothesis that there is no difference between the two types of bearings, under the assumption that both distributions are normal.

   (b) Test the same hypothesis using a non-parametric method.

   (c) Which of the methods – that of part (a) or that of part (b) – do you think is better in this case? (Hint: Think about our assumption in part (a). Does it seem correct?)

2. The presence of an environmental factor is thought to increase the incidence of a disease, so a scientist determines the presence or absence of the environmental factor and of the disease for a randomly selected group of people. Based on the data, does it make a difference? Use a test of significance level 0.05. [The table of the χ^2 CDF on the opposite side of the page may be useful.]

	not present	present
no disease	128	7
disease	19	7

3. [13.29] Suppose a company wishes to examine the relationship of gender to job satisfaction, grouping job satisfaction into four categories: very satisfied, somewhat satisfied, somewhat dissatisfied, and very dissatisfied. The company plans to ask the opinions of 100 employees. Should you, the company's statistician, carry out a chi-squared test of independence or homogeneity?

4. [13.28] In a chi-squared test of homogeneity, would the results change if instead of counts, the entries were proportions of the column total, with each column summing to 1? What if the entries were percentages of the column total?

5. [11.19] An experiment is planned to compare the mean μ_X of a control group to the mean μ_Y of an independent sample of a group given a treatment. Suppose that the observations are approximately normally distributed and that the standard deviation of a single measurement in either group is $\sigma = 5$. There are $n = 25$ samples in each group.

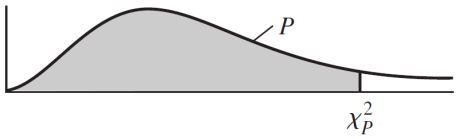
   (a) What is the standard error of the difference of the sample means, $\overline{Y} - \overline{X}$?

   (b) With a significance level $\alpha = 0.05$, what is the rejection region of the test of the null hypothesis $H_0 : \mu_Y = \mu_X$ versus the alternative $H_A : \mu_Y > \mu_X$?

   (c) What is the rejection region of the level $\alpha = 0.05$ test if the alternative is $H_A : \mu_Y \neq \mu_X$?

6. [11.24] Find the exact null distribution of the Mann-Whitney test statistic T for $m = 3, n = 2$.

TABLE 3 Percentiles of the χ^2 Distribution—Values of χ^2_P Corresponding to P



df	$\chi^2_{.005}$	$\chi^2_{.01}$	$\chi^2_{.025}$	$\chi^2_{.05}$	$\chi^2_{.10}$	$\chi^2_{.90}$	$\chi^2_{.95}$	$\chi^2_{.975}$	$\chi^2_{.99}$	$\chi^2_{.995}$
1	.000039	.00016	.00098	.0039	.0158	2.71	3.84	5.02	6.63	7.88
2	.0100	.0201	.0506	.1026	.2107	4.61	5.99	7.38	9.21	10.60
3	.0717	.115	.216	.352	.584	6.25	7.81	9.35	11.34	12.84
4	.207	.297	.484	.711	1.064	7.78	9.49	11.14	13.28	14.86
5	.412	.554	.831	1.15	1.61	9.24	11.07	12.83	15.09	16.75
6	.676	.872	1.24	1.64	2.20	10.64	12.59	14.45	16.81	18.55
7	.989	1.24	1.69	2.17	2.83	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	15.99	18.31	20.48	23.21	25.19